Excerpt from 'Multi-criteria decision analysis for use in transport decision making', DTU Transport Compendium Series part 2, 2014.

The Analytic Hierarchy Process (AHP)

The Analytic Hierarchy Process (AHP), developed by Saaty (1977), is essentially the formalisation of our intuitive understanding of a complex problem using a hierarchical structure as stated in (Hwang and Yoon, 1995). The AHP offers an alternative approach to SMART when a decision maker is faced with a problem involving multiple objectives. The method widely applied to decision problems in areas such as economics and planning, and because the AHP involves a relative complex mathematical procedure, user friendly computer software, such as Expert Choice, has been developed to support the method.

The crux of the AHP is to enable a decision maker to structure a Multi-Attribute Decision Making (MADM) problem visually in form of an attribute hierarchy. An attribute hierarchy has at least three levels: the focus or the overall goal of the problem on the top level, multiple criteria that define alternatives in the middle level, and competing alternatives in the bottom level. When criteria are highly abstract such as e.g. "well-being", sub-criteria (or sub-sub-criteria) are generated subsequently through a multilevel hierarchy.

For example consider the problem of choosing which alternative to construct in a case, where an old road connection through a city has run out of capacity and a new improved connection is needed. The decision makers can choose between: A new by-pass road on the northern side of the city (A_1), an upgrade of the existing connection (A_2), or a new by-pass road on the southern side of the city (A_3). Figure 0.1 shows the generated decision criteria by means of a hierarchical structure.

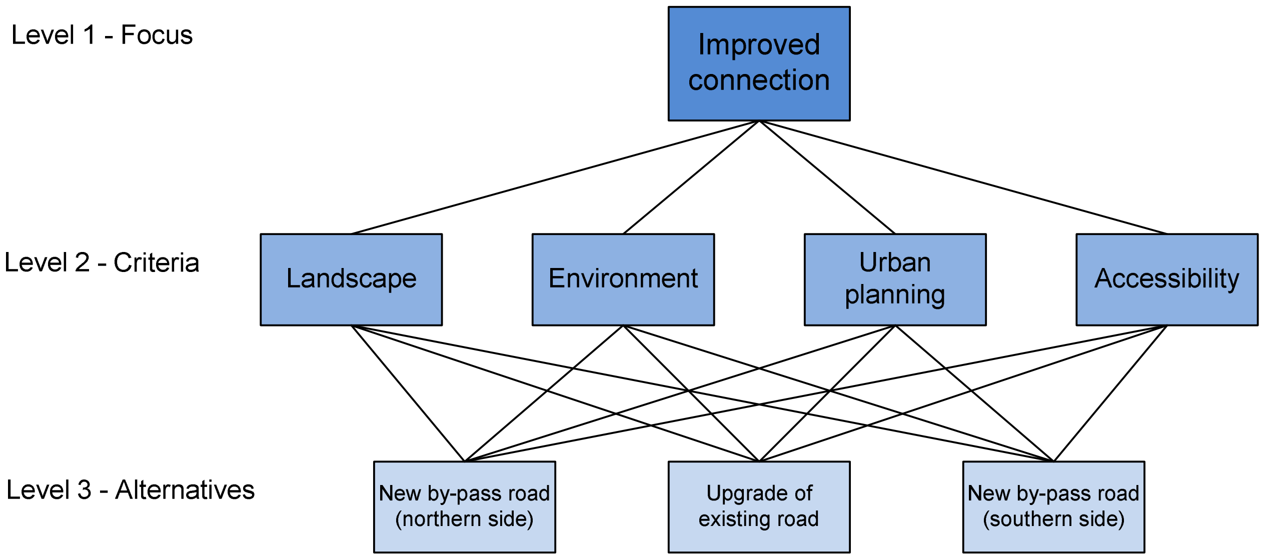

Figure 0.1: A hierarchy for choice of an improved connection

At level 1 the focus is overall an improved road connection. Level 2 compromises the criteria that contribute to the decision making: Landscape (L), environment (E), urban planning (UP) and accessibility (AC). Level 3 consist of the three solution possibilities: A_1, A_2 and A_3. It is obvious that each criterion in level

2 should contribute differently to the focus. The decision can be made on the relative importance among four criteria by pair-wise comparisons, due to the fact that pair-wise comparisons are much easier to make than a comparison of four criteria simultaneously.

In order to help the decision maker to assess the pair-wise comparisons, Saaty created a nine point intensity scale of importance between two elements (Saaty, 2001). The suggested numbers to express degree of preference between the two elements A and B are shown in Table 0.1.

To decide the relative weightings between *n* alternatives, it is in principle only necessary to perform *n-1* assessments. By performing a complete set of full pair-wise comparisons more information than necessary is collected, but a more varied evaluation is obtained, and if one or more answers are inaccurate the other answers will compensate the inaccuracy. The number of judgments, *J*, that have to be made in a full pair-wise comparison can be determined by (Belton and Stewart, 2002):

$$J = \frac{n \cdot (n-1)}{2}$$

Table 0.1: The fundamental scale for pair-wise comparisons (Saaty, 2001)

Intensity of importance	Definition	Explanation
1	Same	Neither of the two alternatives is preferable over the other
3	Weak	One alternative is preferred slightly over the other
5	Clear	One alternative is preferred clearly over the other
7	Strong	One alternative is preferred strongly over the other
9	Very Strong	One alternative is preferred very strongly over the other
2, 4, 6, 8	Compromise	Can be used for graduation between evaluation
Reciprocals of above	If activity i has one of the above nonzero numbers assigned to it when compared with activity j, then j has the reciprocal value when compared with i	A comparison mandated by choosing the smaller element as the unit to estimate the larger one as a multiple of that unit

In the road problem, there are four criteria in level 2. The decision maker then makes six pair-wise judgments among four criteria with respect to level 1 (4(4-1)/2=6):

(L : E) = (7 : 1)   (L : UP) = (1 : 1)   (L : AC) = (7 : 1)
(E : UP) = (1 : 3)   (E : AC) = (2 : 1)   (UP : AC) = (5 : 1)

This information can be concisely contained in a so-called comparison matrix whose element at row *i* and column *j* is the ratio of row *i* and column *j* (Hwang and Yoon, 1995). The comparison matrix *A*, as introduced by Saaty, is seen below:

$$
A = \begin{bmatrix} \frac{w_1}{w_1} & \frac{w_1}{w_2} & . & . & . & \frac{w_1}{w_n} \\ \frac{w_2}{w_1} & \frac{w_2}{w_2} & . & . & . & \frac{w_2}{w_n} \\ . & . & & & & . \\ . & . & & & & . \\ . & . & & & & . \\ \frac{w_n}{w_1} & \frac{w_n}{w_2} & . & . & . & \frac{w_n}{w_n} \end{bmatrix}
$$

Where $w_1, w_2, \ldots, w_n$ is the weights obtained by the comparisons. Applied on the case example that is:

$$
\begin{matrix} L \\ E \\ UP \\ AC \end{matrix} \begin{bmatrix} 1 & L/E & L/UP & L/AC \\ E/L & 1 & E/UP & E/AC \\ UP/L & UP/L & 1 & UP/AC \\ AC/L & AC/L & AC/UP & 1 \end{bmatrix} = \begin{matrix} L \\ E \\ UP \\ AC \end{matrix} \begin{bmatrix} 1 & 7 & 1 & 7 \\ 1/7 & 1 & 1/3 & 2 \\ 1 & 3 & 1 & 5 \\ 1/7 & 1/2 & 1/5 & 1 \end{bmatrix}
$$

The next step for the decision maker is to make pair-wise comparisons of the three alternatives in level 3 with respect to four criteria in level 2:

For L:

$$
\begin{matrix} & A_1 & A_2 & A_3 \\ A_1 & 1 & 1/3 & 2 \\ A_2 & 3 & 1 & 5 \\ A_3 & 1/2 & 1/5 & 1 \end{matrix}
$$

For E:

$$
\begin{matrix} & A_1 & A_2 & A_3 \\ A_1 & 1 & 3 & 1/5 \\ A_2 & 1/3 & 1 & 1/7 \\ A_3 & 5 & 7 & 1 \end{matrix}
$$

For UP:

$$
\begin{matrix} & A_1 & A_2 & A_3 \\ A_1 & 1 & 1/5 & 2 \\ A_2 & 5 & 1 & 7 \\ A_3 & 1/2 & 1/7 & 1 \end{matrix}
$$

For AC:

$$
\begin{matrix} & A_1 & A_2 & A_3 \\ A_1 & 1 & 1/3 & 1/5 \\ A_2 & 3 & 1 & 1/3 \\ A_3 & 5 & 3 & 1 \end{matrix}
$$

After the construction of the pair-wise comparison matrix, the next step is to retrieve the weights of each element in the matrix. There are several methods for retrieving these weights: the originally introduced eigenvector method (Hwang and Yoon, 1981), and the later introduced geometric mean method (Saaty, 2001).

The geometric mean method with calculations regarding the case example is introduced below as this method is the most suitable for calculations in hand. The eigenvector method is described next, but only for a small numerical example as this method uses more demanding calculations that normally will be carried through in a software program such as Expert Choice.

The eigenvector method

The first step in the eigenvector method is to reduce the pair-wise comparison matrix to a comparison vector, i.e. a set of scores (or partial values) representing the relative performance of each alternative. The values in the pair-wise comparison matrix are interpreted as ratios of these underlying scores.

Saaty originally introduced a method of scaling ratios using the principle eigenvector of a positive pair-wise comparison matrix. The method presumes that matrix *A* is (Hwang and Yoon, 1981):

$$A = \begin{bmatrix} a_{11} & a_{12} & . & . & . & a_{1n} \\ a_{21} & a_{22} & . & . & . & a_{2n} \\ . & . & & & & . \\ . & . & & & & . \\ . & . & & & & . \\ a_{n1} & a_{n2} & . & . & . & a_{nn} \end{bmatrix} = \begin{bmatrix} \frac{w_1}{w_1} & \frac{w_1}{w_2} & . & . & . & \frac{w_1}{w_n} \\ \frac{w_2}{w_1} & \frac{w_2}{w_2} & . & . & . & \frac{w_2}{w_n} \\ . & . & & & & . \\ . & . & & & & . \\ . & . & & & & . \\ \frac{w_n}{w_1} & \frac{w_n}{w_2} & . & . & . & \frac{w_n}{w_n} \end{bmatrix}$$

This is a reciprocal matrix (as before), which has all positive elements and has the reciprocal property:

$$a_{ij} = \frac{1}{a_{ji}} \quad \text{and} \quad a_{ij} = \frac{a_{ik}}{a_{jk}}$$

Multiplying A by $\underline{w} = (w_1, w_2, ..., w_n)^T$ yields

$$A \cdot \underline{w} = \begin{bmatrix} \frac{w_1}{w_1} & \frac{w_1}{w_2} & . & . & . & \frac{w_1}{w_n} \\ \frac{w_2}{w_1} & \frac{w_2}{w_2} & . & . & . & \frac{w_2}{w_n} \\ . & . & & & & . \\ . & . & & & & . \\ . & . & & & & . \\ \frac{w_n}{w_1} & \frac{w_n}{w_2} & . & . & . & \frac{w_n}{w_n} \end{bmatrix} \cdot \begin{bmatrix} w_1 \\ w_2 \\ . \\ . \\ . \\ w_n \end{bmatrix} = n \cdot \begin{bmatrix} w_1 \\ w_2 \\ . \\ . \\ . \\ w_n \end{bmatrix} = n \cdot \underline{w}$$

or

$$(A - nI) \cdot \underline{w} = 0$$

Due to the consistency, the system of homogeneous linear equations has only trivial solutions. In general the precise values of w_i / w_j are unknown and must be estimated, so in other words, human judgments cannot be so accurate that the equation can be satisfied completely. In any matrix small perturbations in the coefficients imply small perturbations in the eigenvalues (Hwang and Yoon, 1981). If we define A^{-} as the decision makers estimate of A and $\underline{w}^{-}$ is corresponding to A^{-}, then

$$A^{-}\underline{w}^{-} = \lambda_{\max}\underline{w}^{-}$$

Where $\lambda_{\max}$ is the largest eigenvalue of A^{-}. $\underline{w}^{-}$ can be obtained by solving the system of linear equations. In order to show the steps in the computation of weights, a numerical example is reviewed.

The following example is taken from (Hwang and Yoon, 1981). The positive pair-wise comparison matrix is given:

$$A = \begin{bmatrix} 1 & 1/3 & 1/2 \\ 3 & 1 & 3 \\ 2 & 1/3 & 1 \end{bmatrix}$$

The determinant of $(A - \lambda \cdot I)$ is then set to zero:

$$\det(A - \lambda \cdot I) = \begin{bmatrix} 1-\lambda & 1/3 & 1/2 \\ 3 & 1-\lambda & 3 \\ 2 & 1/3 & 1-\lambda \end{bmatrix} = 0$$

The largest eigenvalue of A, $\lambda_{\max}$, is 3.0536, and we have:

$$\begin{bmatrix} -2.0536 & 1/3 & 1/2 \\ 3 & -2.0536 & 3 \\ 2 & 1/3 & -2.0536 \end{bmatrix} \cdot \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = 0$$

The solution of the homogeneous system of linear equations, where it is assumed that $\sum_{i=1}^{3} w_i = 1$, gives:

$$\underline{w}^{T} = \begin{bmatrix} 0.1571 \\ 0.5936 \\ 0.2493 \end{bmatrix}$$

The geometric mean method

For an approximation method that provides sufficiently close results in most situations, (Saaty, 2001) suggest the geometric mean of a row: Multiply the *n* elements in each row, take the *n*th root, and prepare a new column for the resulting numbers, then normalise the new column (i.e., divide each number by the sum of the numbers). The weights for the four criteria in the case example are shown below.

$$
\begin{matrix} L \\ E \\ UP \\ AC \end{matrix}
\begin{bmatrix}
(1\cdot 7\cdot 1\cdot 7)^{1/4} = & 2.65 \\
(1/7\cdot 1\cdot 1/3\cdot 2)^{1/4} = & 0.56 \\
(1\cdot 3\cdot 1\cdot 5)^{1/4} = & 1.97 \\
(1/7\cdot 1/2\cdot 1/5\cdot 1)^{1/4} = & 0.35
\end{bmatrix}
=
\begin{bmatrix} 0.48 \\ 0.10 \\ 0.36 \\ 0.06 \end{bmatrix}
$$

$$
sum \qquad 5.53 \qquad 1.00
$$

Similarly, the relative contributions (i.e., weights) among three alternatives towards the four criteria are computed below.

$$
\begin{matrix} & L & E & UP & AC \end{matrix}
$$

$$
\begin{matrix} A_1 \\ A_2 \\ A_3 \end{matrix}
\begin{bmatrix}
0.23 & 0.19 & 0.17 & 0.10 \\
0.65 & 0.08 & 0.74 & 0.26 \\
0.12 & 0.73 & 0.09 & 0.64
\end{bmatrix}
$$

The final stage of the AHP is to compute the contribution of each alternative to the overall goal (i.e., improved connection) by aggregating the resulting weights vertically. The overall priority for each alternative is obtained by summing the product of the criteria weight and the contribution of the alternative, with respect to that criterion. Refer to Figure 0.2 for the road choice problem.

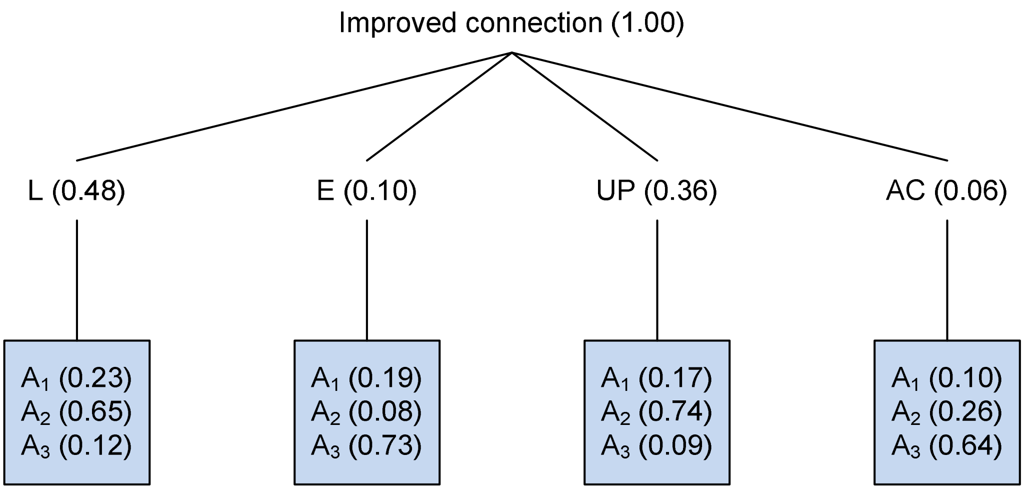

Figure 0.2: Priorities for each hierarchical level

The computation of the overall priority for alternative A1 is as follows

$$
0.48\cdot(0.23)+0.10\cdot(0.19)+0.36\cdot(0.17)+0.06\cdot(0.10)=0.1966
$$

Similarly, they are 0.6020 and 0.2014 for A_2 and A_3 respectively. Therefore the decision maker's choice would be to make an upgrade of the existing road (A_2), if the decision was only to be based on these criteria.

Consistency

The AHP allows inconsistency, but provides a measure of the inconsistency in each set of judgments. This measure is an important by-product of the process of deriving priorities based on pair-wise comparisons. It is natural for people to want to be consistent, as being consistent is often thought of as a prerequisite to clear thinking. However the real world is hardly ever perfectly consistent and we can learn new things only by allowing for some inconsistency with what we already know.

Some causes for inconsistency are listed below:

- *Lack of information.* If the decision maker has little or no information about the factors being compared, then the judgments will appear to be random and a high consistency ratio will result. It is useful to find out that a lack of information exists, although sometimes decision maker might be willing to proceed without immediately spending time and money gathering additional information in order to ascertain if the additional information is likely to have a significant impact on the decision.
- *Lack of concentration.* Lack of concentration during the judgment process can happen if the decision makers become fatigued or are not really interested in the decision. It is the decision analyst job to prevent this from happening.
- *Real world is not always consistent.* The real world is rarely perfectly consistent and is sometimes fairly inconsistent. Football is a good example: It is not uncommon for team A to defeat team B, after which team B defeats team C, after which team C defeats team A. Inconsistencies such as this may be explained as being due to random fluctuations, or to underlying causes (such as match-ups of personnel), or to a combination. Regardless of the reasons, real world inconsistencies do exist and thus will appear in our judgments.
- *Inadequate model structure.* A final cause of inconsistency is “inadequate” model structure. Ideally one would structure a complex decision in a hierarchical fashion such that factors at any level are comparable, within an order of magnitude or so, of other factors at that level. Practical considerations might preclude such a structuring and it is still possible to get meaningful results. Suppose for example, several items that differed by as much as two orders of magnitude were compared. One might erroneous conclude that the AHP scale is incapable of capturing the differences since the scale ranges from 1 to 9. However, because the resulting priorities are based on second, third and higher order dominances, AHP can produce priorities far beyond any order of magnitude. A higher than usual inconsistency ratio will result because of the extreme judgments necessary.

It is important that a low inconsistency ratio does not become the goal of the decision making process. A low inconsistency ratio is necessary but not sufficient for a good decision. It is possible to be perfectly consistent but consistently wrong. It is more important to be accurate than consistent.

Consistency ratio

The consistency ratio is computed from the eigenvalue, $\lambda_{\max}$, which will often turn out to be larger than the value describing a fully consistent matrix. In order to provide a measure of severity of this deviation, Saaty defined a measure of consistency, or consistency index (CI) by (Belton and Stewart, 2002):

$$CI = \frac{\text{Pricipal eigenvalue - size of matrix}}{\text{size of matrix - 1}} = \frac{\lambda_{\max} - n}{n-1}$$

The consistency index is compared to a value derived by generating random reciprocal matrices of the same size, to give a consistency ratio (CR) which is meant to have the same interpretation no matter the size of the matrix. The comparative values from random matrices are as follows in Table 0.2 for 3 ≤ n ≤ 10 (Belton and Stewart, 2002, p. 156):

Table 0.2: Comparative values

Size of matrix	3	4	5	6	7	8	9	10
Comparative value	0.52	0.89	1.11	1.25	1.35	1.40	1.45	1.49

In the small example concerning the eigenvalue method the largest eigenvalue was 3.0536, the consistency index is thus (3.0536-3)/2=0.0268. Since n=3, the consistency ratio is 0.0268/0.52=0.05. A consistency ratio of 0.1 or less is generally stated to be acceptable.

As seen the AHP method is fundamentally different to other assessment methods in many respects. This section is considering strengths of the AHP, but also looks on some of the criticisms which have been made of the technique.

Relative strengths of AHP

Formal structuring of the problem. Like SMART and other decision analysis techniques the AHP provides a formal structure to problems. This allows complex problems to be decomposed into sets of simpler judgments and provides a documented rationale for the choice of a particular option.

Simplicity of Pair-wise comparisons. The use of pair-wise comparisons means that the decision maker can focus, in turn, on each small part of the problem. Only two attributes or options have to be considered at any one time so that the decision maker's judgmental task is simplified. Verbal comparisons are also likely to be preferred by decision makers who have difficulty in expressing their judgments numerically.

Redundancy allows consistency to be checked. The AHP requires more judgments to be made by the decision maker than is needed to establish a set of weights. For example, if a decision maker indicates that attribute A is twice as important as B, and B, in turn, is three times as important as C, then it can be inferred that A is six times more important than C. However, by also asking the decision maker to compare A with C it is possible to check the consistency of the judgments. It is considered to be good practice in decision analysis to obtain an input for a decision model by asking for it in several ways and then asking the decision maker to reflect on any inconsistencies in the judgments put forward. In the AHP this is carried out automatically (Goodwin and Wright, 2009).

Versatility. The wide range of applications of the AHP is evidence of its versatility. In addition to judgments about importance and preference, the AHP also allows judgments about the relative likelihood of events to be made. This has allowed it to be applied to problems involving uncertainty and also to be used in forecasting. AHP models have also been used to construct scenarios by taking into account the likely behaviour and relative importance of key actors and their interaction with political, technological, environmental, economic and social factors (Goodwin and Wright, 2009).

Criticism of AHP

Conversion from verbal to numeric scale. Decision makers using the verbal method of comparison will have their judgments automatically converted to the numerical scale, but the correspondence between the two scales is based on untested assumptions. If you indicate that A is weakly more important than B the AHP will assume that you consider A to be tree times more important, but this may not be the case.

Inconsistencies imposed by 1 to 9 scale. In some problems the restriction of pair-wise comparisons to a 1 to 9 scale is bound to force inconsistencies on the decision maker. For example if A is considered to be 5 times more important than B, and B is 5 times more important than C, then to be consistent A should be judged to be 25 times more important than C, however this is not possible.

Meaningfulness of responses to questions. Unlike SMART, weights are elicited in the AHP without reference to the scales on which attributes are measured. For example, a person using SMART to choose a house might be asked to compare the value of reducing the daily journey to work from 100 kilometers to 10 kilometers with the value of increasing the number of bedrooms in the house from two to four. Implicit in this type of comparison is the notion of a trade-off or exchange: 90 fewer kilometers may only be half as valuable as two extra bedrooms. AHP questions, which simply ask for the relative importance of attributes without reference to their scales, are therefore less well defined, if they are meaningful at all. This fuzziness may mean that the questions are interpreted in different, and possibly erroneous, ways by decision makers.

New alternatives can reverse the rank of existing alternatives. This issue, which is related to the last point, has attracted much attention. Suppose that you are using the AHP to choose a location for a new company and the weights you obtained from the method give the following order of preference: 1. London, 2. Paris and 3. Rome. However before making the decision you discover that a site in Berlin is also worth considering, so you repeat the AHP to include this new option. Even though you leave the relative importance of the attributes unchanged, the new analysis gives the following rankings: 1. Paris, 2. London, 3. Berlin and 4. Rome, so the rank of Paris and London have been reversed, which does not seem to be intuitive reasonable. (Goodwin and Wright, 2009) claims that this arises from the way in which the AHP normalizes the weights to sum to 1, and that this is consistent with a definition of weights which is at variance with that used in SMART. Most decision makers would consider SMART to be the reasonable one.

Number of comparisons required may be large. While the redundancy build into the AHP is an advantage, it may also require a large number of judgments from the decision maker. Consider, for example, an office location problem which involves 8 alternatives and 8 attributes, this would involve 224 pair-wise comparisons of importance or preference. This requirement to answer a large number of questions can reduce the attraction of the AHP in the eyes of potential users, even though the questions themselves are considered to be easy.

The axioms of the method. In (Goodwin and Wright, 2009) it is argued, that the SMART method is well founded on a set of axioms, that is, a set of rules which are intended to provide the basis for a rational decision making. The clarity and intuitive meaning of these axioms allows their appeal, as rules for rational behaviour to be debated and empirically tested. In contrast to this (Goodwin and Wright, 2009) argues that the axioms of the AHP are not founded on testable descriptions of rational behaviour.

Consistency checks

It is good practice to carry out more than the minimum number of comparisons necessary to specify the set of criteria weights, thus building in a check of consistency of the decision makers' judgments. As with scores, the assessment of weights is also implicitly a process of pair wise comparisons. This may be formalised by specifying a reference criterion against which all others are compared (requiring the minimum number of comparisons), or each criterion may be compared with every other one giving full specification (requiring n(n-1)/2 comparisons) as in the AHP approach. Alternatively something between these two extremes may be sought by judicious choice of criteria to be compared.

Working with weak information

The process of determining values for criteria weights calls for a lot of hard thinking on the part of the decision maker. Questions such as those described above are difficult to answer. Depending on the circumstances of the decision, an alternative way of proceeding might be to use the rank order of to give simple initial estimates of criteria weights and then to use this as starting point for extensive sensitivity analysis. This may show that the preferred alternative is insensitive to changes in weights which preserve rank order – in which case it would not be necessary to specify more precise values. Or it may indicate that attention should be focussed on the weight assigned to a specific criterion.

Some writers (e.g. Edwards and Barron (1994) discussing their SMARTER approach) suggest that, in the presence of only ordinal information, an initial analysis can be carried out using weights which are in some sense most central in the region defined by $w_1 > w_2 > w_3 > \ldots > w_n > 0$. One possibility is to estimate weights by the centroid, i.e. the arithmetical average of the extreme points of the region. When normalising the weights to sum to 1, it is easily confirmed that the n extreme points are: (1, 0, 0, …, 0), (1/2, 1/2, 0, …, 0), (1/3, 1/3, 1/3, 0, …, 0), …, (1/n, 1/n, …, 1/n). However, the use of the centroid to generate a set of weights is not entirely satisfactory, as it leads to rather extreme values. For example, with n = 3 criteria, the centroid weights are 0.611, 0.278 and 0.111 respectively, so that the ratio of w_1 to w_3 is 5.5, so that the third criteria will only have a very marginal influence on the outcome, contrary to what appears to be meant by the inclusion of all 3 criteria in the analysis. The situation becomes more extreme as n increases; with n = 5, the centroid weights are 0.457, 0.257, 0.157, 0.090 and 0.039. The ratio from w_1 to w_5 is now over 11:1, so that the fifth criterion has an almost vanishingly small influence. Experience seems to suggest, that at weight ratio of 5 or more indicates an almost absolute dominance of one criterion over another, so that larger ratios should not really be expected amongst criteria which have been retained in the value tree.

Two possibilities for ameliorating the extreme effects of using the centroid may be (Belton and Stewart, 2002):

- Inclusion of a further constraint to the effect that no criterion i will have been included in the model if the ratio of w_1 (the largest weight) to w_i exceeds some factor R: For example with R = 9, the extreme points of the feasible weight region for n = 3 will be: (9/11, 1/11, 1/11), (9/19, 9/19, 1/19) and (1/3, 1/3, 1/3). The centroid weights based on these extremes are 0.541, 0.299 and 0.159.
- Assume a geometrically decreasing set of weights, with each w_i being a constant proportion r of the next most important weight w_{i-1}: The centroid weights do in fact decrease in an approximately geometric fashion, but with a high rate of decrease (with a value of r around 0.4 – 0.45 for n = 3, and around 0.5 – 0.55 for n = 5). Some anecdotal experience suggests a rather less dramatic rate of reduction in weights, certainly well above 0.5. For example, even with r increased up to 0.6, the estimated weights for n = 3 are much more moderate, namely: 0.510, 0.306 and 0.184.

Overall evaluation

The overall evaluation of an alternative is determined by first multiplying its value score on each bottom-level criterion by the cumulative weight of that criterion and then adding the resultant values. If the values relating to individual criteria have been assessed on a 0 to 100 scale and the weights are normalised to sum to 1 then the overall values will lie on a 0 to 100 scale. If the criteria are structured as a value tree then it is also informative to determining scores at intermediate levels of the tree. In the example from earlier this allows the alternatives to be compared, for example, on accessibility.

However, the determination of an overall value should by no means be viewed as the end of the analysis, but simply another step in furthering understanding and promoting discussion about the problem. Although the underlying model is simple and static that should not be a limitation in its use. It provides a powerful vehicle for reflecting back to decision makers the information they have provided, the judgments they have made, and an initial attempt at synthesising these. The extent to which the model will be a successful catalyst for discussion of the problem and for learning about one's own and other's values, depends on the effectiveness with which feedback can be provided. Simple, static visual displays are an effective means of reflecting back information provided and well-designed visual interactive interfaces provide a powerful vehicle for exploring the implications of uncertainty about values.

In exploring the model decision makers should test the overall evaluation and partial aggregations of information against their intuitive judgment. Are the results in keeping with intuition? If not, why not? Could some values have been wrongly assessed? Is there an aspect of performance which is not captured in the model? Is the additive model inappropriate? Or does the model cause the decision makers to revise their intuitive judgments? The aim of the analysis should be to arrive at convergence between the results of the model and the decision makers' intuition.

Decision makers should look not only at the overall evaluation of alternatives, but at their profiles. How is an alternative's overall value made up? Is it a good "all-rounder" or does it have certain strengths and weaknesses? Alternatives with similar overall scores can have very different profiles. Is there any dominating, or dominated alternatives? One option dominates another if it does at least as well on all criteria relevant to the decision. In simple terms, if there is an option which dominates all others it should be preferred, or if an option is dominated by another it should not be a candidate for choice. However, rather than acting as rigid guidelines, these concepts should be used as catalysts for further thought and learning about the problem situation.