Risk Analysis and Monte Carlo Simulation within Transport Appraisal

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Abstract: This small report contains an easy step-by-step approach to Quantitative Risk Analysis (RA) and Monte Carlo simulation (MCS). The report is in no way a full review on how to perform a RA and MCS nor intended to be a complete and rigorous treatment of probability and statistics. Thus it has been avoided to give a formal examination of the theory concerning statistics and probability in which case the report instead tries to present a more functional description of RA and MCS. However, some statistical terms considered when applying probability distribution functions are discussed. A special emphasis has been put on the separation between inherent randomness in the modeling system and the lack of knowledge. These two concepts have been defined in terms of variability (ontological uncertainty) and uncertainty (epistemic uncertainty). Furthermore, a consideration about which probability distributions to make use of and where to implement them are explained. Finally, the eventual outcome from the RA and MCS is described together with a further perspective of the CBA-DK framework model.

1. Introduction

The main objective of risk analysis is to establish a rational foundation for objective decision making. The risk analysis aims at quantifying the undesirable effects that a given activity may impose on humans, environment or economical values. The objective of the decision process is then to identify the solution that in some sense minimizes the risk of the considered activity (Friis-Hansen, 2005).

Although the word “risk” often is used in daily speech the meaning of the word seem to be relative imprecise. When we talk about risk related to a given activity, everybody has intuitively an understanding of what is meant: the higher the risk the more dangerous the activity. It is important to notice that the word “risk” is only used when there is a chance, but not certainty, that something undesirable may happen. We may talk about risk of a car accident, risk of loosing money on the stock market, etc.; but we do not talk about the risk of winning the national lottery, although winning is uncertain, it is not undesirable. Similarly, we do not talk about the risk of paying taxes, though not liking it there is no uncertainty involved - sadly. The risk and uncertainties are key features of most business and government problems and needs to be assessed before any decisions rational or not is applied the business. The concept of uncertainties has been around for several decades e.g. Sir Francis Bacon, Philosopher (1561-1626) states:

If we begin with certainties, we shall end in doubts; but if we begin in doubts, and are patient in them, we shall end in certainties.

When assessing any decision support problems it is important to bear in mind, that any results stemming from the Quantitative Risk Analysis (RA) is not finalized results but merely a help to the decision-makers. The term risk analysis as used in this report pertains to the evaluation of the total uncertainty associated with benefit-cost estimates. Traditional risk analysis is often referred to in economical or investment studies where a risk of a certain outcome is incorporated in the further calculations. In our sense risk analysis is a part of the sensitivity analysis performed as uncertainty evolves within various data, parameter or impact assessment.

A complete risk assessment procedure is likely to consist of five steps (Vose, 2002):

1. Identification of the risk that is to be analyzed
2. A qualitative description of the problem and the risk – why it might occur, what you can do to reduce the risk, probability of the occurrence etc.
3. A quantitative analysis of the risk and the associated risk management options that is available to determine or find an optimal strategy for controlling and hereby solving the risk problem
4. Implementing the approved risk management strategy
5. Communicating the decision and its basis to various decision-makers.

The essence of the traditional risk analysis approach is to give the decision-maker a mean by which he can look ahead to the totality of any future outcome. The advantage of using any risk analysis approach is the possibility of differentiating the feature of risk information in terms of outcome criteria such as Net Present Value (NPV), the Internal rate of Return (IRR) or the Benefit/Cost rate (B/C-Rate) by probability distributions (Hertz & Thomas, 1984).

1.1 Quantitative Risk Analysis

Single point or deterministic modeling involves using a best guess estimate concerning each variable within the model to determine the actual outcome. Hereby, the uncertainty or actual sensitivity is performed on the model to determine how much such an outcome might vary from the point estimate calculated earlier. These variations are often referred to as “what if” scenarios where the advantage of using Quantitative Risk Analysis is that instead of only creating a number of possible scenarios it effectively accounts for every possible value that each variable within the model can take by use of various *continuous* probability distributions. Each variable/parameter assigned a probability distribution result in different scenarios that are weight together by the probability of occurrence.

The main structure of a RA model is somewhat very similar to a deterministic single value rate of return model except that each variable in the RA model is represented by a probability distribution function (PDF). The objective is to calculate the combined impact of the *variability* and *uncertainty* in the models parameters in order to determine a total probability distribution of the model. The resulting point estimate is then transformed into an interval estimate illustrated in terms of a probability distribution. The technique used in the following work is a Monte Carlo simulation (MCS) which involves a random sampling method concerning each different probability distribution selected for the actual model set-up. As these distributions are defined hundreds or even thousands of different scenarios can be produced in the following, these types of scenarios are referred to as *iterations*. Each probability distribution is sampled in a manner such that it reproduces the original shape of the distribution meaning that the actual model outcome reflects the probability of the values occurrence.

1.2 Uncertainty and Variability

The human striving of predicting a future outcome has been a wanted skill for many decades. Uncertainty and variability satisfies our inability to be able to precisely predict the future meaning that if we are able to determine these two components we would be able to predict the future outcome.

Sir David Cox defines the two concepts as: v*ariability is a phenomenon in the physical world to be measured, analyzed and where appropriate explained. By contrast, uncertainty is an aspect of knowledge (Vose, 2002 pp 18).*

The following describes briefly the two concepts of variability and uncertainty. For further information about these concepts please refer to Vose (2002) or Salling & Leleur (2006).

Variability uncertainty (Ontological)

Many empirical quantities (measurable properties of the real-world systems being modelled) vary over space or time in a manner that is beyond control, simply due to the nature of the phenomena involved. The concept ontology arises from philosophy which is the study of being or existence. Basically, it seeks to exploit the conceptions of reality with one single question: “What is there?” (Wikipedia, 2007).

Variability uncertainty (in the further only named: variability) is then the effect of a given chance and is a function of the given model system. This variable is the most difficult to explain as it is not possible to reduce by study or further measurements. However, it is possible to minimize the variability by changing the physical modeling system. One of the best known case examples of a variable experiment is the tossing of a coin. The normal prediction of a coin toss is a probability 50% for heads and 50% for tails. However, when making the experiment it is not possible to predict whether you achieve heads or tails in the next toss due to the coins inherent randomness.

It is therefore impossible to get a zero contribution from variability as designing any modeling systems. However, as mentioned earlier there are possibilities of controlling the variability by altering the whole modeling system. Different sources of variability can be distinguished e.g. inherent randomness of nature: the chaotic and unpredictable nature of natural processes and human behaviour (behavioural variability): ‘non-rational’ behaviour, discrepancies between what people say and what they actually do (cognitive dissonance) etc. see Figure 1.

Uncertainty (Epistemic)

The epistemic uncertainty (in the further named: uncertainty) is related to many aspects of modelling and policy analysis – e.g., limited and inaccurate data, measurement error, incomplete knowledge, limited understanding, imperfect models, subjective judgement, ambiguities, etc. The term epistemology stems from philosophy and is the defined as the theory of knowledge and belief. Basically, the epistemic thinking seeks to exploit the question: “What do people know?” (Wikipedia, 2007).

Uncertainty is then the modelers’ lack of knowledge concerning the parameters that characterizes the modeling system (defined as the level of ignorance). This factor has the possibility of being reduced by further studies or measurements. Uncertainty is further by definition subjective since it is a function of the modelers’ level of knowledge; however, some techniques are available to allow a certain degree of objectiveness.

A schematically overview of the uncertainty concept are illustrated in Figure 1.

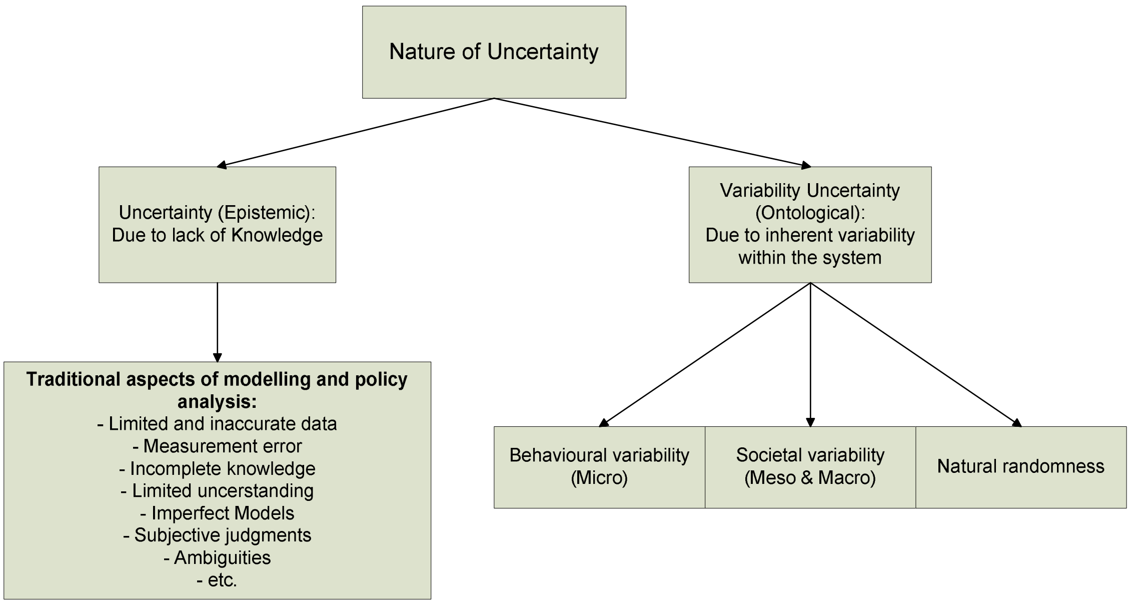


Figure 1. The nature of Uncertainty: Inherent variability or lack of knowledge (Salling & Leleur, 2006)

A common mistake is failure to distinguish between the uncertainty inherent in sampling from a known frequency distribution (variability), and the uncertainty that arises from incomplete scientific or technical knowledge (uncertainty). For example, in throwing a fair coin, one knows that the outcome will be heads 1/2 the time, but one cannot predict what specific value the next throw will have (variability). In case that the coin is not fair, there will also be epistemic uncertainty, concerning the frequency of the heads.

Intuitively is a separation of the two terms not easy to comprehend as they both share exactly the same probability distributions looking and behaving identically. A reasonable assumption is therefore to create the same Monte Carlo model just dividing the different uncertain and variable parameters with different distributions. This is, however, likely to give wrongful information of the simulation, as the model outcome is represented in a resultant single distribution. This distribution represents the "best guess" distribution in terms of a composition between the uncertainty and the variability parameters. In this sense the interpretation of the modeling result is difficult due to the scaling of the vertical axis. This probability scale is a combination of both components resulting in ignorance in determination both of the inherent randomness of the system and what component is due to our ignorance of the same system (Salling, 2006).

One of the main advantages of separating the uncertainty and variability is that the total uncertainty of a model system does not show the actual *source* of the uncertainty. The information corresponding to the two sources implied in the total uncertainty is of great relevance towards the decision makers in a given situation. If a result shows that the level of uncertainty in a problem is huge this means that it is possible to collect further information and thereby reduce the level of uncertainty which enables us to improve our estimate. On the other hand, if the total uncertainty is nearly all due to variability it is proven to be a waste of time to collect further information and the only way to improve and hereby reduce the total uncertainty would be to change the whole modeling system.

1.3 Monte Carlo Simulation (MCS)

The term "Monte Carlo" was first introduced by *von Neumann* and *Ulam* during World War II, as code name for the secret work at Los Alamos where the allied forces combined tried to discover the atom bomb. It was suggested by the gambling casinos at the city of Monte Carlo in Monaco to get some "advertising". The work at Los Alamos involved direct simulation of behavior concerned with random neutron diffusion in fissionable material.

The Monte Carlo method is now one of the most powerful and commonly used techniques for analyzing complex problems. The different types of applications can be found in many fields from radiation transport to river basin modeling. Furthermore, it is not only on stochastic processes the MCS is applicable, but also at deterministic problems this method is usable. There are three major points suggesting a Monte Carlo method instead of traditional simulation methods:

1. In the Monte Carlo method time does not play as substantial role as it does in stochastic simulation in general
2. The observations in the Monte Carlo method, as a rule, are independent. In simulation, however, the experiment with the observations is over time so, as a rule, these are serially correlated and hence dependent of each other.
3. In the Monte Carlo method it is possible to express the responses in a rather straightforward manor by simple functions of the stochastic input variables. In simulation the response is often a very complicated one and can only be expressed explicitly by computer programs.

The nice thing about MCS is that it is easy to apply. When you combine several uncertain values, determining the uncertainty on the result can be very complex. For example, when you add two uncertain values, the uncertainty on the result is somewhat less that the sum of the original uncertainties. Using MCS, this and similar effects are handled automatically so you don't need to know much about statistics to get accurate results.

One of the key factors when creating a simulation model is to see whether the model is evolving over time. Whether the problem is deterministic or stochastic the MCS has proven useful. Stochastic simulation is actually a statistical sampling method with the actual model or in statistical circles a design analysis. Because sampling from a particular distribution involves the use of random numbers, these types of simulations is named MCS. The Monte Carlo method was original considered to be a technique using random or pseudorandom numbers chosen from a uniformly distributed interval [0;1] (Law & Kelton, 2000).

The principle behind MCS made use of in the framework of CTT, the so-called CBA-DK model, is briefly illustrated in Figure 2, which shows the principles behind simulation of investment planning.

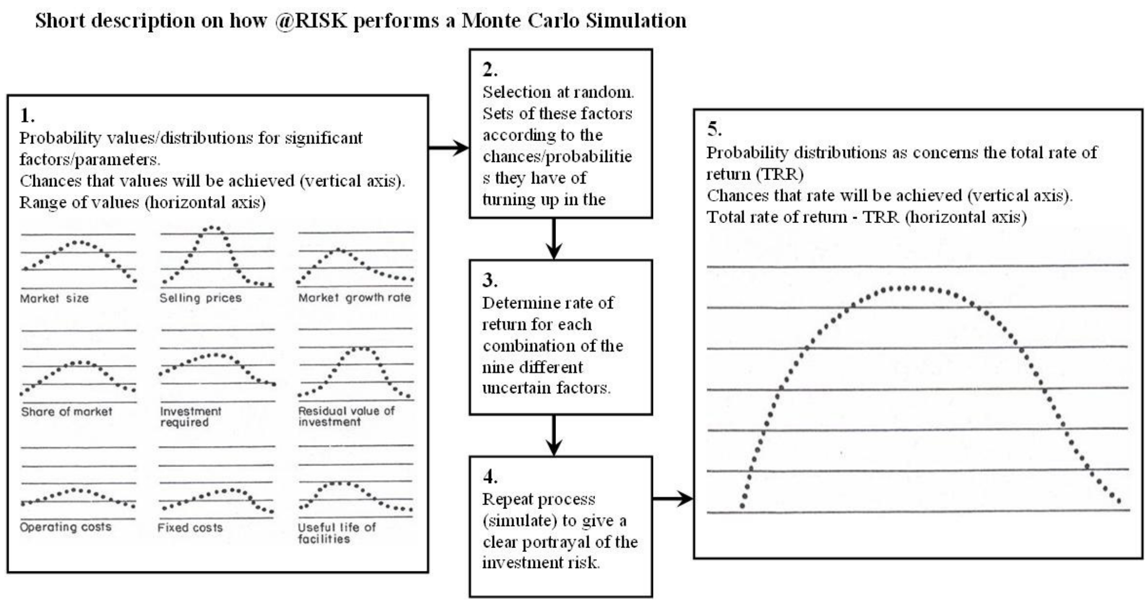


Figure 2. Simulation principles of an example from investment strategies moderated from (Hertz & Thomas, 1984)

In the above case there are 9 different uncertain parameters:

1. Market size
2. Selling prices
3. Market growth rate
4. Share of market
5. Investment required
6. Residual value of investment

7. Operating costs
8. Fixed costs
9. Useful life of facilities

Each parameter is assigned a probability distribution as indicated. By using Monte Carlo simulation it is possible to 'join' all 9 distributions in one 'result'-distribution, which is the case in the last step (here a rate of return).

It is now possible to give a short 7-step overview of how a Monte Carlo simulation works containing two possible methods of sampling: Monte Carlo sampling and Latin Hypercube sampling (step 3).

Step 1: Determine the uncertain parameters/values
Step 2: Add a suitable probability distribution to each selected parameter
Step 3: Generate for each individual distribution a random value by a sampling method
Step 4: For each iteration (step 1 illustrated in Figure 2) a new benefit/cost-rate is calculated
Step 5: Repeat the process (Step 2 to 4) by a relatively large amount of iterations
Step 6: It is now possible to view a most likely B/C-rate combined with the interval of the lowest and the highest found B/C-rate
Step 7: Plot and view the new total probability distribution of the B/C-rate (@RISK is able to give a histogram or accumulated probability curves)

1.4 Choosing the Probability Distribution

The probability distributions are in the further all continuous distributions hence they will be referred to as distributions only. The five types in use and applicable towards our framework scenarios are the Uniform, Triangular, PERT, Normal and Erlang distribution. Within each small paragraph an impact concerning transport appraisal has been implemented. The research in the application of various probability distributions is an ongoing topic in the Ph.D. study entitled *Decision Support and Risk Assessment for Transportation Projects*.

To give a brief overview of the various impacts together with the assignment of probability distributions and their nature of uncertainty, see Figure 3.

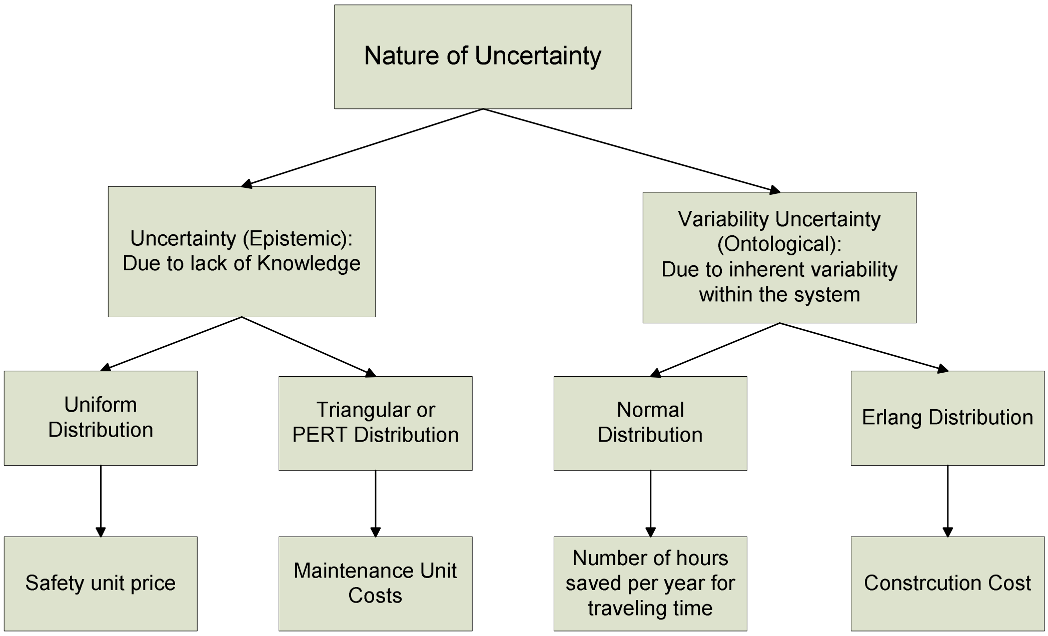


Figure 3. Overview of applied uncertain impacts within the Risk Analysis Framework (Salling & Leleur, 2006)

The above mentioned distribution functions are all related to road infrastructure projects thus a more elaborate description of e.g. air - or sea transport projects are handled in other reports or papers among others (Salling & Leleur, 2007).

1.4.1 The Uniform probability distribution

The simplest form of a distribution set is formed on basis of the uniform distribution. A uniform distribution is one for which the probability of occurrence is the same for all values of *X*. It is sometimes called a rectangular distribution. For example, if a fair die is thrown, the probability of obtaining any one of the six possible outcomes is 1/6. Since all outcomes are equally probable, the distribution is uniform. The uniform distribution is a generalization of the rectangle function because of the shape of its probability density function (PDF). It is parameterized by the smallest and largest values that the uniformly-distributed random variable can take, *a* and *b*. The PDF of the uniform distribution is thus:

$$f(x) = \begin{cases} \dfrac{1}{b-a} & For\ a < x < b, \\ 0 & For\ x < a\ or\ x > b \end{cases}$$

The typical uniform distribution is sketched as shown in Figure 4.

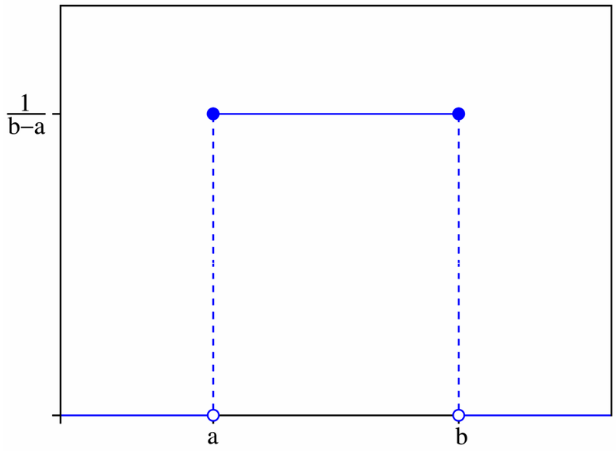


Figure 4. Illustration of a uniform distribution within the interval of [a ; b]

There have been some misunderstandings or perhaps difference of opinion in literature around the terms *mean*, *mode* and *median*. The *mean* value is often referred as the μ-value, and defined via two terminologies the **expected value** (population mean) and **the average** (sample mean). The expected value is defined as the sum of the probability of each possible outcome of the experiment multiplied by its payoff ("value"). Thus, it represents the average amount one "expects" to win per bet if bets with identical odds are repeated many times. Note that the value itself may not be expected in the general sense; it may be unlikely or even impossible. A game or situation in which the expected value for the player is zero (no net gain nor loss) is called a "fair game."

The average is in statistics often referred to as arithmetic mean which is to distinguish from the geometric mean[i] and the harmonic mean[ii]. The mean of the Uniform distribution or better explained the average is $\frac{a+b}{2}$ which easily can be seen from Figure 4. For a data set, the mean is just the sum of all the observations divided by the number of observations. Once we have chosen this method of describing the communality of a data set, we usually use the standard deviation to describe how the observations differ. The standard deviation is the square root of the average of squared deviations from the mean.

In probability theory and statistics, the *median* is a number that separates the higher half of a sample from the lower half. It is the **middle value** in a distribution, above and below which lie an equal number of values. This states that half of the sample will have values less than or equal to the median and half of the sample will have values equal to or greater than the median.

To find the *median* of a finite list of numbers, arrange all the observations from lowest value to highest value and pick the middle one. If there is an even number of observations, one often takes the mean of the two middle values. As concerns the Uniform distribution it is clear that the range of samples will lie between a and b with an equal probability which means that the median = mean $= \frac{a+b}{2}$. The mode will be explained in the following section concerning the Triangular distribution.

The benefits or costs stemming from the accidents are determined by their value towards the society stemming from multiplying the expected number of accidents saved with a societal unit price. By estimating material costs such as car damage, policy costs etc. with personal and social costs e.g. loss of production, hospital costs etc. a monetary unit is derived. The Danish methodology is accounting for 9 various unit costs per traffic accident which contributes to the overall uncertainty of this impact, see Table 1.

	Reported traffic accident	Reported traffic accident with personal injury	Reported personal injury
Cost related to personal injury	374	876	674
Cost related to material loss	476	1,115	858
Cost related to the society (loss in production)	264	620	477
Total costs	**1.115**	**2.611**	**2.009**

Table 1. Various unit costs for traffic accidents in 1,000 DKK per accident in price level 2003 (DMT, 2006)

Then dependent on the road type a total amount of personal injuries can be determined calibrated on a 1 km section of the given road. The Uniform distribution shows the assumed uncertainty included in the price-setting where information on a high and low range is estimated. In this case run an estimate with ± 10% to the standard unit price has been applied.

1.4.2 The Triangular probability distribution

The Triangular distribution is typically used as a subjective description of a population for which there is only limited sample data. It is based on knowledge of the minimum and maximum and an inspired guess (referred to as the Most Likely value ML - *mode*) as to what the modal value might be, see Figure 5. Despite being a simplistic description of a population, it is a very useful distribution for modeling processes where the relationship between variables is known, but data is scarce. The Triangular distribution has a lower limit a, mode c and upper limit b.

$$f(x|a,b,c)=\begin{cases}\dfrac{2\cdot(x-a)}{(b-a)\cdot(c-a)} & For\ a\le x\le b,\\[2ex] \dfrac{2\cdot(b-x)}{(b-a)\cdot(b-c)} & For\ c<x\le b\end{cases}$$

The triangular distribution or in an enhanced version; the *Trigen*-distribution, allows the upper and lower boundaries to be skewed as illustrated in the upper part of the PDF. The *Trigen*-distribution further offers the analyst the possibility of choosing a confidence interval, where the upper and lower boundaries can be exceeded within a predefined percentage. An illustration of the triangular distribution is shown in Figure 5.

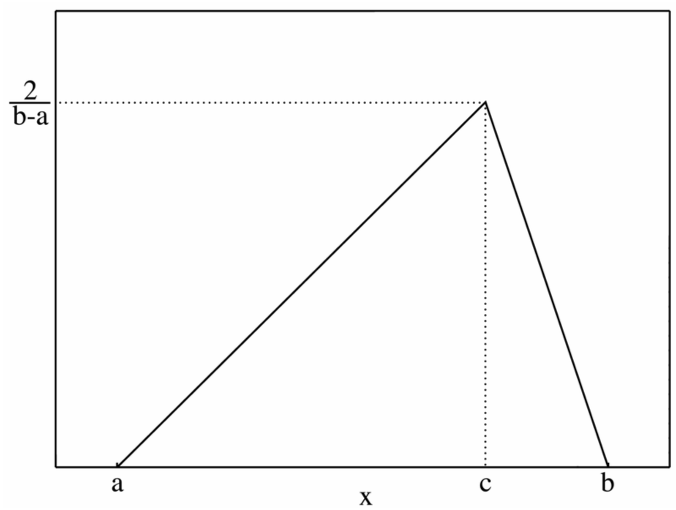


Figure 5. Illustration of a triangular distribution within the interval of [a ; b] with the mode c

The *mode*, as illustrated in Figure 5 by *c*, is the value that has the largest number of observations, namely **the most frequent value** within a particular set of values. For example, the mode of {1, 3, 6, 6, 6, 7, 7, 12, 12, 17} is 6. The mode is not necessarily unique, unlike the arithmetic mean. It is especially useful when the values or observations are not numeric since the mean and median may not be well defined. For example, the mode of {apple, apple, banana, orange, orange, orange, peach} is orange.

In a Gaussian (i.e. bell curve) distribution, the mode is at the peak just as the Triangular distribution. The mode is therefore the value that is *most* representative of the distribution. For example, if you measure people's height or weight and the values form a bell curve distribution, the peak of this bell curve would be the most common height or weight among these people. The mean/average value can often be influenced by outliers or skews and can be well away from the peak of the distribution and therefore not a value representative of the largest number of people. The mode for the uniform distribution is actually any value in the interval of [*a*;*b*].

The mean of the triangular distribution is defined as $\frac{a+b+c}{3}$ whereas the median is defined by

$$\begin{cases} a+\dfrac{\sqrt{(b-a)\cdot(c-a)}}{\sqrt{2}} & \text{for } c>\dfrac{b-a}{2} \\ b-\dfrac{\sqrt{(b-a)\cdot(b-c)}}{\sqrt{2}} & \phantom{\text{for }} c\le\dfrac{b-a}{2} \end{cases}$$

A more comprehensive distribution to implement instead of the Triangular distribution is the Beta-PERT distribution.

1.4.3 The Beta distribution (PERT)

The Beta-PERT distribution (from here on just referred to as the PERT distribution) is a useful tool for modeling expert data. The PERT stands for Program Evaluation and Review Technique and stems from 1958 where it was assigned a so-called schedule procedure. The PERT is derived from the Beta distribution which mathematically is fairly simple and furthermore covers a huge variety of types of skewness. When used in a Monte Carlo simulation, the PERT distribution can be used to identify risks in project and cost models especially based on the resemblance to the Triangular Distribution. As with any probability distribution, the usefulness of the PERT distribution is limited by the quality of the inputs: the better your expert estimates, the better results you can derive from a simulation.

Like the triangular distribution, the PERT distribution emphasizes the "most likely" value over the minimum and maximum estimates. However, unlike the triangular distribution the PERT distribution constructs a smooth curve which places progressively more emphasis on values around (near) the most likely value, in favor of values around the edges. In practice, this means that we "trust" the estimate for the most likely value, and we believe that even if it is not exactly accurate (as estimates seldom are), we have an expectation that the resulting value will be close to that estimate, see Figure 6.

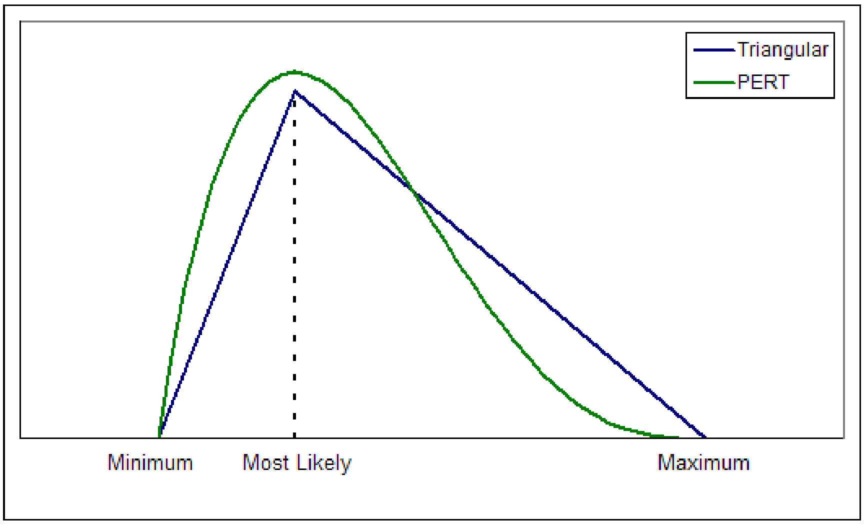


Figure 6. Illustration of the PERT distribution and the comparison with a Triangular distribution

The advantage of using a PERT distribution is to be seen from the differences in their mean values i.e. $Mean_{Triang} = \frac{Min + Mode + Max}{3}$ vs. $Mean_{PERT} = \frac{Min + 4 \cdot Mode + Max}{6}$. The average of all three parameters in the PERT distribution has got four time the weighting on the Mode. In real-life problems we are usually capable of giving a more confident guess at the mode than the extreme values hence the PERT distribution brings a much smoother description of the tales of the impacts to be considered.

The maintenance costs (MC) are developed based on empirical accounting formulas considering different cost factors (Leleur, 2000 p. 158). The modeling scheme of determining MC has been found by analyzing previous expenditures together with the road type, average daily traffic and the width of the lanes. Furthermore, it has been found suitable to use a Triangular distribution to illustrate the uncertainty (Salling 2006). Specifically, the uncertainty assigned to this parameter using the Triangular distribution is defined by 10% possibility of achieving a lower MC (min.), the most likely value is the previously calculated MC and 50% possibility of achieving a higher value at the tales (max.). It should be noted that this effect normally is a disbenefit towards society due to the fact that new infrastructure projects tend to be more expensive in maintenance than prior.

1.4.4 The Normal probability distribution

The Normal distribution is an extremely important probability distribution in many fields. It is a family of distributions of the same general form, differing in their *location* and *scale* parameters: the mean and standard deviation, respectively. The standard normal distribution is the normal distribution with a mean of zero and a standard deviation of one (the green curve in Figure 7). It is often called the bell curve because the graph of its probability density resembles a bell.

The probability density function of the normal distribution with mean μ and variance σ^2 (equivalently, standard deviation σ) is an example of a Gaussian function,

$$f(x;\mu,\sigma) = \frac{1}{\sigma \cdot \sqrt{2\pi}} \cdot \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

If a random variable X has this distribution, we write $X \sim N(\mu , \sigma^2)$. If $\mu = 0$ and $\sigma = 1$, the distribution is called the *standard normal distribution* and the probability density function reduces to, $f(x) = \frac{1}{\sqrt{2\pi}} \cdot \exp\left(-\frac{x^2}{2}\right)$.

Some different types of probability density function of the normal distribution with various parameter values are shown in Figure 7.

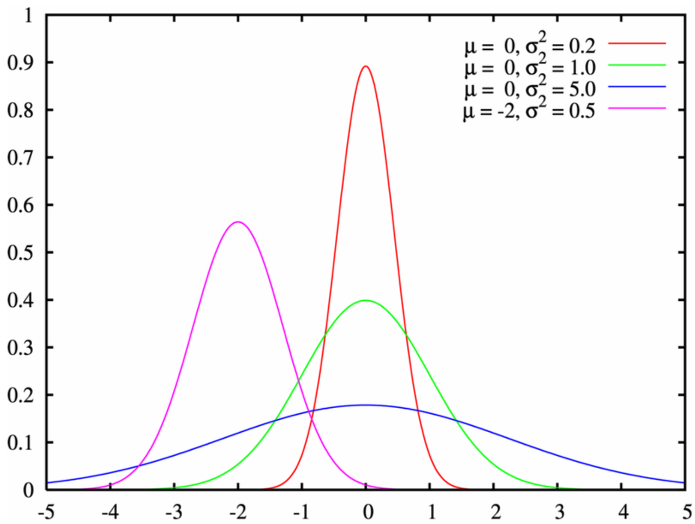


Figure 7. Illustration of a normal distribution with mean μ and std. deviation σ (Wikipedia, 2006)

Some of the most notable qualities of a Normal distribution are that it is symmetric around the mean and the mean is also both the mode and median value.

One of the most obvious advantages of constructing new or improved infrastructure is that it results in time savings both towards people and freight, traveling between areas served by the new or improved infrastructure. In contrast to the construction costs the Travel Time Savings (TTS) normally counts as a benefit towards society. This impact is probably the most important benefit in transport infrastructure projects. Benefits stemming from this category often make up a share in the range from 70-90% of the overall benefits (Leleur 2000 p 108). Subsequent to detailed study of alternative alignments, it will be possible to analyze travel time saving in some detail. However, building traffic models or assignment models includes several pitfalls ensuring uncertainty.

The TTS has been found to follow a Normal distribution where the mean is based upon the net change in hours spent on traveling in the influence area of the road project. Standard deviations relating to traffic models applied in Denmark have been found to be around 10-20% (Knudsen, 2006) and (Leleur et al., 2004). By testing a traffic model in several scenarios it has been proven that the standard error within this model is around 11% for the transport mode and 16% for the traffic loads. Further investigations show that a standard deviation in the area of 10% for smaller projects and 20% for large projects are not unlikely (Ibid.).

1.4.5 The Gamma distribution (Erlang)

The Erlang distribution is a probability distribution with wide applicability primarily due to its relation to the exponential and Gamma distributions. The Erlang distribution was developed by A. K. Erlang[iii] to examine the number of telephone calls which might be made at the same time to the operators of the switching stations. This work on telephone traffic engineering has been expanded to consider waiting times in queuing systems in general. The distribution is now used in the field of stochastic processes.

The Erlang distribution has a positive value for all the numbers greater than zero, and is parameterized by two parameters: the shape k, which is an integer, and the rate λ, which is real. The distribution is sometimes defined using the inverse of the rate parameter, the scale θ, applicable within the software program @RISK. When the shape parameter k equals 1, the distribution simplifies to the exponential distribution the red curve in Figure 8. The Erlang distribution has been found useful in combination with the so-called Lichtenbergs principle in obtaining a mean and a std. deviation from successive calculation (Lichtenberg, 2000).

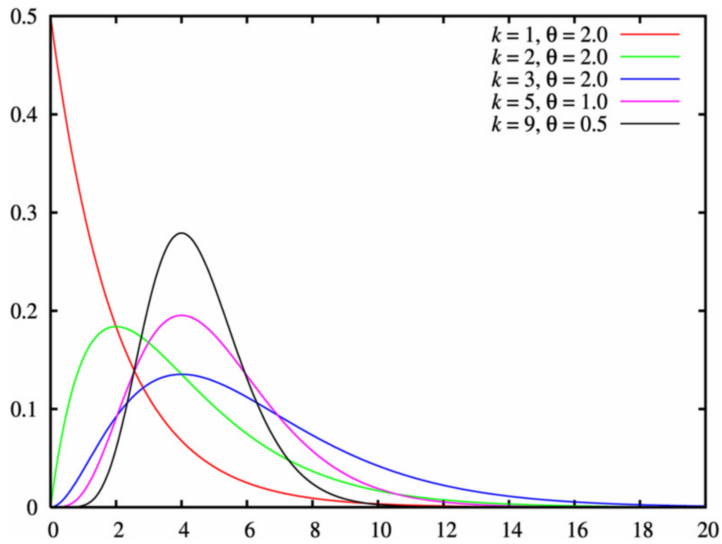


Figure 8. Illustration of an Erlang distribution with various shape and scale parameters (Wikipedia, 2006)

The PDF of the Erlang distribution is $f(x;k,\lambda)=\frac{\lambda^k x^{k-1} e^{-\lambda x}}{(k-1)!}$ for x>0 where e is the base of the natural logarithm and ! is the factorial function. An alternative, but equivalent, parameterization uses the scale parameter θ which is simply the inverse of the rate parameter (i.e. θ = 1 / λ): $f(x;k,\theta)=\frac{x^{k-1} e^{-\frac{x}{\theta}}}{\theta^k (k-1)!}$ for x > 0.

Because of the factorial function in the denominator, the Erlang distribution only defined when the parameter *k* is a positive integer. The mean value can be determined on basis of the shape parameter, as for $k = 1$, the Erlang distribution is similar to the Exponential distribution and if *k* increases the Erlang distribution is similar to the normal distribution. The mean is hence forward k/λ or $k \cdot \theta$ and the mode is defined by $(k-\lambda)/\lambda$ only for $k \geq 1$.

Input parameters towards the Erlang distribution is calculated by use of Lichtenberg's principle taking into account the upper and lower bound together with the most likely value (ML) based upon successive calculation.

A Danish statistician has developed a principle based upon successive calculation (Lichtenberg, 2000). The strength of applying the so-called Lichtenberg principle is that the decision-maker only has to consider a minimum, most likely (ML) and maximum value. It is among others used for several issues including support, optimize and estimating budget allowances especially within the construction area. Some other key areas where the principle has been applied are strategic planning and risk analysis. Then by use of a so-called triple estimation approach the mean and standard deviation are calculated by the two following formulas (Lichtenberg, 2000 p. 125):

$$\mu=\frac{(\min.+2.9\cdot ML+\max.)}{4.9} \qquad (1)$$

$$s = \frac{|\max. - \min.|}{4.65} \qquad (2)$$

The properties of the Erlang distribution requires a shape (k) and a scale (θ) parameter. From the triple estimation is the mean (μ) calculated by (1). The relationship to the scale parameter is found by the equation: $\theta = \frac{\mu}{k}$. The applicability of the Erlang distribution is related to the variation of the scale parameter.

The family of Erlang functions is thus a generalization of the exponential function (describing the "function of life") known from e.g. biological science and the reliability area. In fact the Erlang function with $k = 1$ is identical to the exponential function (hereby the illustration of lifespan methodology due to the extremely skewed distribution).

One of the key effects and probably the one with the highest overall impact on an appraisal study is the construction cost, at least in the preliminary phase of any transport infrastructure project. In order for the road authorities or government to prepare reliable financial road programs the necessity for accurate estimates of future funding are vital. Future funding is obviously never known as they are dependent on shifting governments etc. The cost of investing or perhaps determining which projects to invest in ex-ante is often underestimated normally explained by e.g. technical problems, delays, etc. Some authors even think that construction costs in the field collectively are underestimated in the planning phase (Wilmot & Cheng, 2003) & (Flyvbjerg et al., 2003). Other explanations of the general underestimation are the dynamical way an infrastructure project is developing over time. In the pre-face you normally look upon traditional impacts of building e.g. a new road. However, most often during the project new and better choices are made for instance in noise precautions, a new alignment of the road etc. These costs are off course not possible to take into account in advance. The decision-makers also tend to change their preferences during the course of action – especially in large-scale projects. These non-quantifiable preferences are often not taken into account in the preliminary phase which makes the overall construction cost more expensive than originally estimated.

Within the construction of road infrastructure projects in Denmark forecasting future construction costs has been achieved basically by constructing a unit rate e.g. DKK per kilometer highway of a predefined road type (Lahrmann & Leleur, 1997). This method is, however, in many circles considered unreliable due to site conditions such as typography, in situ soil, land prices, environment, traffic loads vary sufficiently from location to location etc. (Wilmot & Cheng, 2003). During literature it is therefore clear that estimating construction costs during infrastructure appraisal has assigned a relatively high degree of uncertainty. The following shows a way to illustrate the uncertainty through probability distributions. Finally in this section, a proposal of a confidence interval and an extensive data collection of large scale construction cost estimations are shown.

Four bullet points for estimating construction costs with probability distributions have been proposed in (Back et al. 2000).

- Upper and lower limits which ensures that the analyst is relatively certain values does not exceed. Consequently, a **closed-ended** distribution is desirable.
- The distribution must be **continuous**
- The distribution will be **unimodal**; presenting a most likely value
- The distribution must be able to have a greater freedom to be higher than lower with respect to the estimation – **skewness** must be expected.

Three probability distributions come into mind when looking at the four bullets. The most obvious choice is the triangular distribution or the PERT distribution both satisfying the latter points. However, the authors point out the Gamma distribution as a likely and suitable distribution even though it is not entirely following the first bullet point due to the open ended tail (Back et al., 2000 p. 30 tab. 1).

It has, furthermore, been found that a shape parameter in the range of k = 5-15 matches the distribution of the uncertainty involved in determining the construction cost (Salling, 2006) & (Lichtenberg, 1990). The family of Erlang functions is thus a generalization of the exponential function (describing the "function of life") known from e.g. biological science and the reliability area. In fact the Erlang function with $k = 1$ is identical to the exponential function (hereby the illustration of lifespan methodology due to the extremely skewed distribution). Using $k = 5$ the function resembles a Lognormal distribution which also is highly appropriate when the parameter is a product of many factors. Finally, when $k \geq 10$ the distribution is brought closer to the Gaussian distribution (Normal Distribution) which again is relevant when a cost parameter is the sum of more than one element (Lichtenberg, 2000). The family of Erlang functions, as shown in Figure 8 seems to represent the vast majority of real life uncertainties quit well.

By test it has, however, been shown that a k value ranging from 4-7 does not reveal any significant change in the result. In the following it has been chosen to select a k-value of 5 – further investigations of this value together with other families of the Gamma distribution is to be implemented in the future work.

1.4.6 Overview

This section has described the used probability distributions applied within the decision modeling group framework. This is by far no elaborate investigation of all probability distributions nor intended to. The applied distribution functions are gathered in Table 2 to provide an overview.

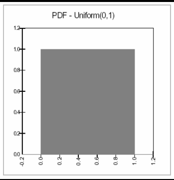
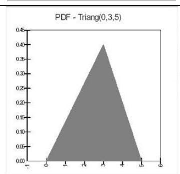
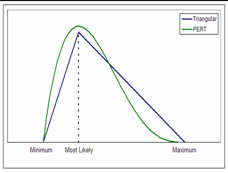
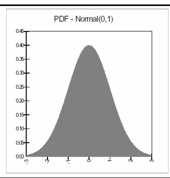
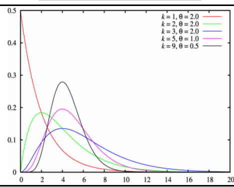
Distribution	Illustration	Properties	Framework
Uniform: Safety unit costs		Close ended Equal probability	Equal possibility of under- and overestimating the most likely value with ± 10%
Triangular: Maintenance Unit costs		Close ended Possibility of skewness	Possibility of respectively under-estimation of ML is [-10% ; ML ; +50%]
Beta (PERT): Maintenance Unit costs		Close ended Possibility of skewness	Possibility of respectively under-estimation of ML is [-10% ; ML ; +50%]
Normal: Travel time savings		Close ended No skewness possible	Most likely value is set to first year impact, std. dev. is set to 15%
Gamma (Erlang): Construction costs		Semi-close ended (open to the right) Possibility of skewness	*k*-value is set to 5 and θ is calculated on basis of the mean from Lichtenberg [-25% ; ML ; +100%]

Table 2. Overview of applied probability distributions in the CBA-DK framework

The set of probability distribution is thus ranging from open ended distributions to close ended distributions applied where needed. The intervals defined are all determined on basis of literature studies and tests conducted in e.g. different master thesis', papers and reports.

1.5 Case Example

The city of Frederikssund is situated in the northern part of Zealand, Denmark (see Figure 9). Roskilde Fjord is separating the peninsula of Hornsherred from Frederikssund and the rest of the Greater Copenhagen area. The bridge – Kronprins Frederiks Bro – at Frederikssund is the only connection across the fjord. The traffic in the area around Frederikssund and Hornsherred is experiencing great problems crossing the bridge. The capacity of the bridge, which was built back in 1935, has been exceeded by far several years ago.

Figure 9 Frederikssund situated in the northern part of Zealand

The current plans concerning the construction of a motorway between Copenhagen and Frederikssund will only increase the traffic problems in the area. In order to solve the problems many different alternatives for a solution have been proposed. However, according to preliminary studies conducted by the city of Frederikssund in cooperation with the road authorities, only four alternatives seem realistic (DMT, 2005):

1. An upgrade of the existing road through Frederikssund and the construction of a new bridge parallel to the old one
2. A new high-level bridge and a new by-pass road south of Frederikssund
3. A short tunnel with embankments and a new by-pass road south of Frederikssund
4. A long tunnel without embankments and a new by-pass road south of Frederikssund

According to the characteristics of the above mentioned alternatives different impacts will be derived from each alternative containing varying investment costs and layouts.

1.5.1 Cost Benefit Approach

The Cost Benefit Analysis (CBA) is used to calculate the socio-economic consequences that the project will result in. Economic profitability is calculated and on the basis of this the four different alternatives are compared. The CBA provides foundation for choosing the most optimal for the society, as the purpose of this type of analysis is to maximise the society's resources. A conventional CBA is carried out in the CBA-DK model in accordance with the Danish Manual for Socio-economic appraisal (DMT, 2003). The CBA includes an assessment of the following principal items:

- Construction and maintenance costs
- Travel time savings and other user benefits
- Accident savings and other external effects
- Taxation
- Scrap value
- Tax distortion

By the use of traffic model calculations the impacts as a result of each of the four alternatives are found and implemented in the CBA-DK model. Forecasting is added to the impacts according to assumptions about the future development in traffic. In accordance with DMT (2006) the Discount Rate is set to 6 %, the Net Taxation Factor is set to 17 % and the Tax Distortion is set to 20 %.

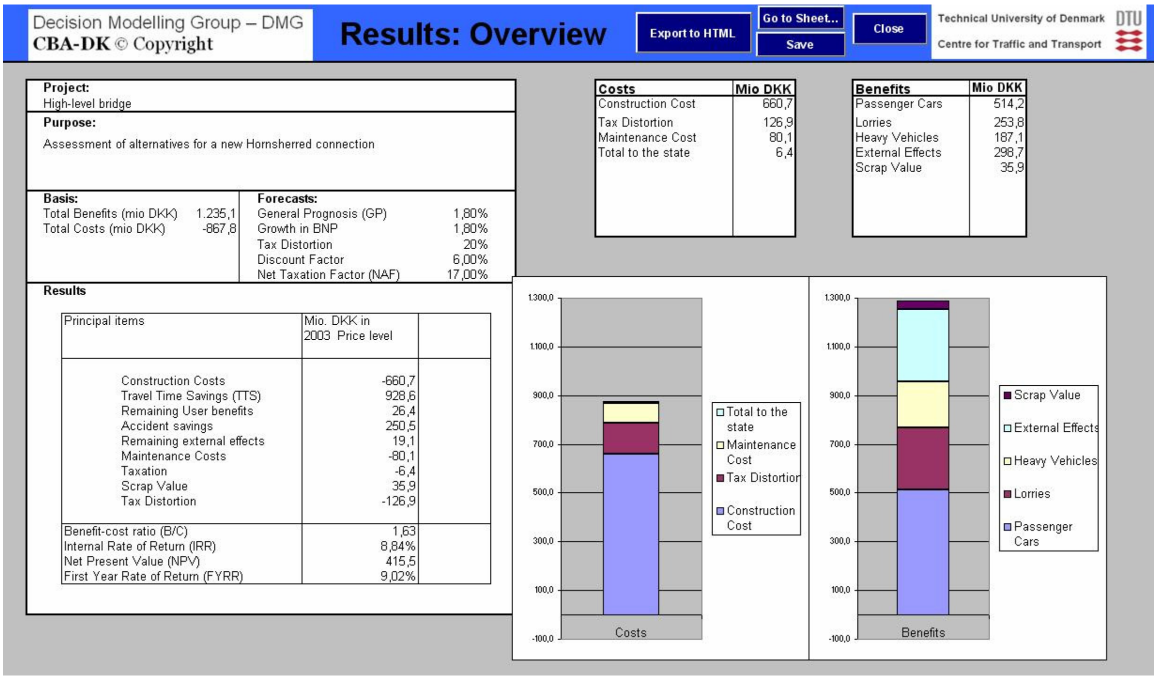


Figure 10 The results sheet in the CBA-DK model (Barfod et al., 2007)

Figure 10 shows an example on how the results are presented in the CBA-DK model. The principal items are listed on the left hand side and two columns depicting respectively the costs and benefits proportional to each other are illustrated in the right hand side. The results from the CBA-DK model for all of the four alternatives are shown in Table 3 in terms of the three previously described investment criteria.

Table 3. Investment criteria of the four different alternatives concerning the Hornsherred case

	B/C-ratio	IRR [%]	NPV [mil DKK]
High-level bridge	**1.63**	**8.84**	**415.5**
Short tunnel	0.99	5.95	-10.2
Long tunnel	0.25	2.06	-1,869.3
Upgrade of connection	0.36	2.76	-235.6

The CBA results clearly show that high-level bridge is the only beneficial alternative if the decision is based solely on monetary impacts. The three other alternatives are not beneficial seen from the investment criteria; the short and the long tunnels because of their high construction costs and the upgrade because of its poor time saving benefits.

1.5.2 The Risk Analysis Approach

The results derived from Figure 10 give a clear identification of the main input variables that have the strongest effect on the overall framework model. It is clear that one of the key impacts is the investment costs (construction costs). By implementing Lichtenbergs principle together with the in Table 2 predefined intervals a Monte Carlo simulation for the construction costs is made. Furthermore, the travel time savings (TTS) are found to be of great relevance for the case of Frederikssund. Implementing a normal distribution where the mean (μ) is set to be the most likely value and a standard deviations relating to traffic models applied in Denmark have been found to be around 10-20% see Table 2. Finally, is the maintenance costs (MC) subjected to simulation where a modified beta distribution is applied to illustrate the uncertainty as described in Table 2.

The feasibility risk study to be applied in the CBA-DK model is made up by simulating 2000 iterations. The possibility of applying e.g. different scenarios, evidently by various input parameters creates varying degrees of uncertainty expressed by the steepness of the descending accumulated graph (Leleur et al., 2004). Rigorous conclusions of the resulting distributions are off course up to the decision-makers to debate, clearly, the input to the probability distributions is subjective of nature.

The purpose of the RA result is to provide the decision-makers with a mean for widen their assessment of the possible B/C-rate (Hertz & Thomas, 1984). Specifically, Figure 11 shows descending accumulated graphs illustrating the “certainty” of achieving a certain B/C-ratio or better. Obtaining a probabilistic view of the B/C-ratio is especially beneficial when several projects are to be evaluated. The possibility of applying, e.g. different scenarios, evidently by various input parameters creates varying degrees of uncertainty expressed by the steepness of the descending accumulated graph (Leleur et al., 2004).

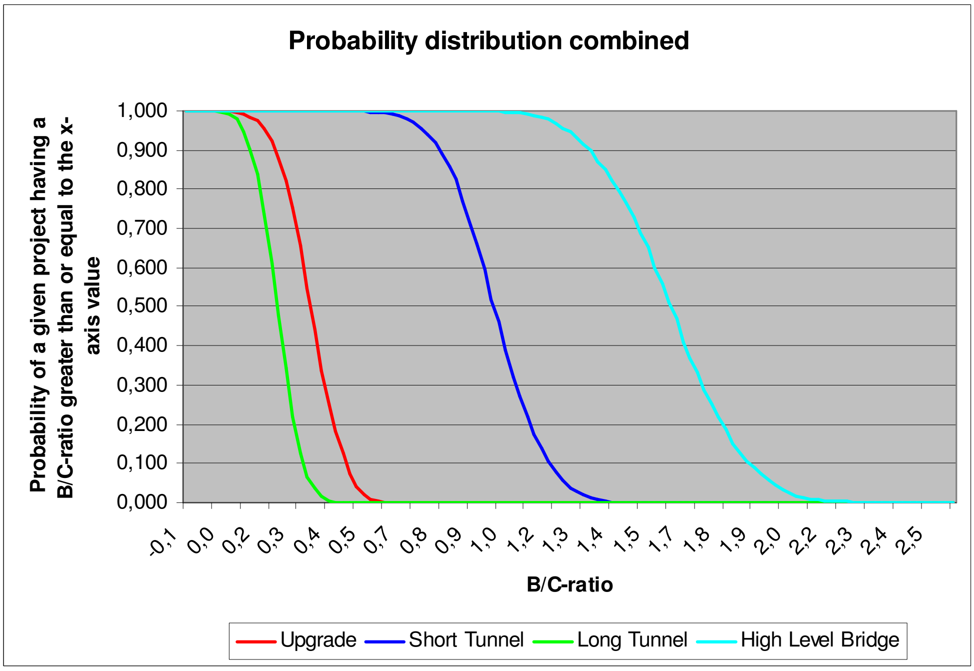


Figure 11. Resulting graphs of the B/C-ratio implementing both Erlang- PERT- and normal distributions (Barfod et al., 2007)

From the results derived from Figure 11 the Long Tunnel project and the Upgrade project clearly returns infeasible projects. The y-axis illustrates the probability that a given project returns a B/C-ratio greater than or equal to the x-axis value illustrating the given B/C-ratio. The Short Tunnel project returns a B/C-ratio above 1.00 in 45% of the iterations. The High Level Bridge Project, however, is the only project that has a B/C-ratio above 1.00 in all of the iterations, which means that if the decision-makers only looked upon the CBA, the High Level Bridge would be the most beneficial project seen from a societal point of view.

Clearly, a simulation accounting for the multi-criteria impacts would increase the decision-makers preferences. However, the sensitivity tests conducted within the CBA-DK model has proved sufficient with varying importance weights for respectively the CBA and MCA parts.

Finally, the feasibility modeling and risk analysis contributes in making decisions easier and hereby more informed e.g. for the politicians. However, since this merely is a decision support system for assisting decision-makers in making the most informed decision, the result with the best socio-economic turnover is not always the political choice. The council in Frederikssund actually prefers the Long Tunnel project which from the CBA and Risk Analysis clearly is the worst projects due to the high construction costs.

1.6 Conclusions

Since most of the simulation models use random variables as input, stated in the previous as randomized probability distributions the simulation output data themselves are random. Care must then be taken in drawing conclusions about the model's true characteristics both concerning the random variables but also the inter correlations. The four chosen impacts used for the MCS are all assumed non-correlated hence no interdependencies are present.

The actual Monte Carlo Simulation is then based upon the two sets of previous mentioned parameters and distributions. The purpose is off course to give the decision-makers a mean to widen their assessment of the possible B/C-rate (Hertz & Thomas, 1984). Obtaining a probabilistic view of the B/C-ratio is especially beneficial when several projects are to be evaluated. The possibility of applying e.g. different scenarios, evidently by various input parameters creates varying degrees of uncertainty expressed by the steepness of the descending accumulated graph (Leleur et al., 2004).

The feasibility risk to be adopted in the actual case is of course up to the decision-makers to debate but the features to deal with uncertainty in the CTT framework may help support their considerations. Some of these will be to get acquainted with the various assumptions behind the scenarios, probability distributions, and the way the latter have been assessed/estimated and related to the different scenarios.

It is increasingly a requirement in model-based decision support that uncertainty has to be communicated in the science-engineering/policy-management interface. During the past decade several significant contributions to concepts, terminology, and typology have been proposed. However, there is no generally accepted approach to communication about uncertainty. The result is confusion and frequent lack of mutual understanding.

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[i] The Geometric Mean of a set of positive data is defines as the n'th root of the product of all the members of the set, where *n* is the number of members. In a formula: the Geometric Mean of $a_1, a_2, ...a_n$ is $(a_1 \cdot a_2 \cdot ....a_n)^{1/n}$, which is $\sqrt[n]{a_1 \cdot a_2 \cdot .... \cdot a_n}$. The Geometric Mean of a data set is always smaller than or equal to the set's mean (the two means are equal if and only if all members of the data set are equal).

[ii] The Harmonic Mean is one of several methods of calculating an average. Typically, it is appropriate for situations when the average of rates I desired. The Harmonic Mean of the positive real numbers $a_1, a_2, ...a_n$ is defined to be $\dfrac{n}{\dfrac{1}{a_1} + \dfrac{1}{a_2} + ... + \dfrac{1}{a_n}}$.

[iii] Agner Krarup Erlang (1878-1929) was a Danish mathematician, statistician and engineer who invented the fields of traffic engineering and queuing theory. Erlang vas born in Lønborg near Tarm in Jutland. He developed his theory concerning telephone traffic over several years.